

# Probability Worksheet

The interface shows a top row of buttons: 'n: 10', 'r: 3', 'nCr', 'nPr', 'RAN#', and 'N!'. Below this is a 'Probability Distribution' section with a 'Select >' button and two dropdown menus. The 'Probability Density' section has buttons for 'p(x)' and 'p(x)<sup>-1</sup>'. The 'Lower-Tail Probability' section has buttons for 'P(x)' and 'P(x)<sup>-1</sup>'.

This worksheet allows the calculations of combinations, permutations, random numbers, factorial and probabilities of selected distributions.

Probability Menu Actions	
[ n ]	Stores the number of total items.
[ r ]	Stores the number of items to be taken at a time.
[ nCr ]	Calculates the number of combinations. $nCr = n! / [ r! \cdot (n - r)! ]$
[ nPr ]	Calculates the number of permutations. $nPr = n! / (n - r)!$
[ RAN# ]	Enters a random number in the range $0 \leq x < 1$
N!	Calculates the factorial of the displayed number.
[ Select ► ] Exponential Normal t-Student Weibull	Select one of the available Probability Distribution.
[ p(x) ]	Calculates the probability density of the displayed number.
[ p(x) <sup>-1</sup> ]	Calculates the inverse probability density of the displayed number.
[ P(x) ]	Calculates the lower-tail cumulative probability of the displayed number.
[ P(x) <sup>-1</sup> ]	Calculates the inverse lower-tail cumulative probability of the displayed number.

## Example: Combinations

Using 10 colored balls, how many different color combinations of three balls can be chosen?

Keystrokes	Description
10 [ n ]	Type the number of total items (10 colored balls).
3 [ r ]	Type the size of the sample (3 balls)
[ nCr ]	Calculate the number of possible combinations. <b>Result = 120.00</b>

## Example: Permutations

Using 5 books labeled A, B, C, D and E, how many different ways can three books be placed on a shelf?

Keystrokes	Description
5 [ n ]	Type the number of total items (5 books).
3 [ r ]	Type the size of the sample (3 books).
[ nPr ]	Calculate the number of possible permutations. <b>Result = 60.00</b>

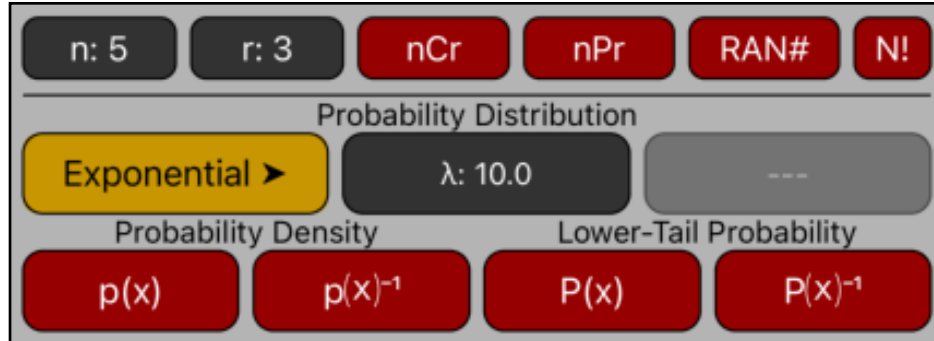
## Example: Random Number Generator

Store a seed value of 42 and generate a sequence of 5 random numbers.

Keystrokes	Description
42 [STO] [RAN#]	Store the initial random seed.
[RAN#]	Generate the 1st random number. <b>Result = 0.7445</b>
[RAN#]	Generate the 2nd random number. <b>Result = 0.3427</b>
[RAN#]	Generate the 3rd random number. <b>Result = 0.1111</b>
[RAN#]	Generate the 4th random number. <b>Result = 0.4223</b>
[RAN#]	Generate the 4th random number. <b>Result = 0.0811</b>

The following examples assumes the “Probability” menu is already visible in the calculator and the display format is set to 6 decimal places.

## Exponential Probability Distribution



When the Exponential probability density function is selected, the distribution “rate parameter” ( $\lambda$ ) can be entered in the corresponding button.

The Probability Density Function is:  $p(x) = \lambda e^{-\lambda x}$

### Example: Exponential Distribution

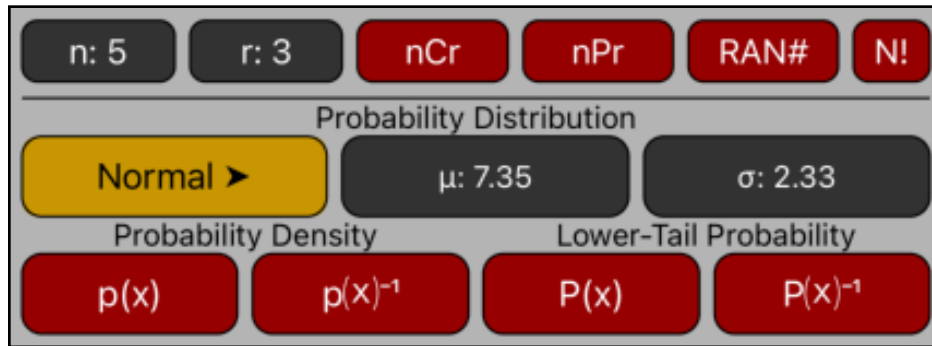
Consider an Exponential random variable with a rate of 10.

1. What is the probability for a value equal to 0.2  $\Rightarrow p(0.2) = ?$
2. If the probability is 5%, what is the value  $\Rightarrow p^{-1}(0.05) = ?$
3. What is the probability of a value  $\leq 0.2$   $\Rightarrow P(x \leq 0.2) = ?$
4. What is the value ‘z’ for probability of  $x \leq z$  is 5%  $\Rightarrow P^{-1}(x \leq z) = 0.05 ?$

### Solution:

Keystrokes	Description
Distribution [ <b>Exponential</b> ▶ ]	Select the Exponential Probability Distribution
10 [ $\lambda$ ]	Type the distribution rate and enter it.
0.2 [ <b>p(x)</b> ]	1) Calculate the probability. <b>Result = 1.353353</b>
0.05 [ <b>p(x)<sup>-1</sup></b> ]	2) Calculate the z-value. <b>Result = 0.529832</b>
0.2 [ <b>P(x)</b> ]	3) Calculate the probability. <b>Result = 0.864665</b>
0.05 [ <b>P(x)<sup>-1</sup></b> ]	4) Calculate the z-value. <b>Result = 0.005129</b>

# Normal Probability Distribution



When the Normal probability density function is selected, the distribution “mean” ( $\mu$ ) and standard deviation ( $\sigma$ ) can be entered in the corresponding buttons.

The Probability Density Function is: 
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

## Example: Normal Distribution

Consider a Normal random variable with a mean of 7.35 and a standard deviation of 2.33.

1. What is the probability for a value equal to 5.35  $\Rightarrow p(5.35) = ?$
2. IF the probability is 5%, what is the value  $\Rightarrow p^{-1}(0.05) = ?$
3. What is the probability of a value  $\leq 5.35$   $\Rightarrow P(x \leq 5.35) = ?$
4. What is the value ‘z’ for probability of  $x \leq z$  is 5%  $\Rightarrow P^{-1}(x \leq z) = 0.05 ?$

## Solution:

Keystrokes	Description
Distribution [ Normal ► ]	Select the Normal Probability Distribution
7.35 [ $\mu$ ], 2.33 [ $\sigma$ ]	Input the distribution mean and standard deviation.
5.35 [ $p(x)$ ]	1) Calculate the probability. <b>Result = 0.118457</b>
0.05 [ $p(x)^{-1}$ ]	2) Calculate the z-value. <b>Result = 11.005837</b>
5.35 [ $P(x)$ ]	3) Calculate the probability. <b>Result = 0.195344</b>
0.05 [ $P(x)^{-1}$ ]	4) Calculate the z-value. <b>Result = 3.517491</b>

# Weibull Probability Distribution

n: 5	r: 3	nCr	nPr	RAN#	N!
Probability Distribution					
Weibull ▶		k: 20.0	λ: 100.0		
Probability Density			Lower-Tail Probability		
p(x)	p(x) <sup>-1</sup>	P(x)	P(x) <sup>-1</sup>		

When the Weibull probability density function is selected, the distribution “shape” parameter (k) and the “scale” parameter (λ) can be entered in the corresponding buttons.

The Probability Density Function is: 
$$p(x) = \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-(x/\lambda)^k}$$

### Example: Weibull Distribution

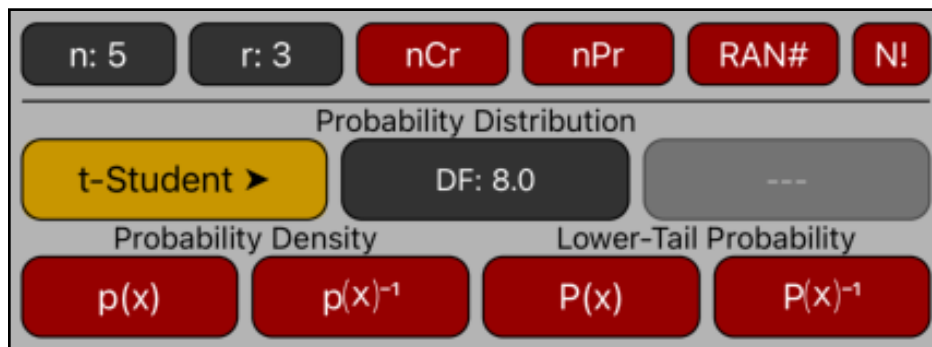
Consider a Weibull random variable with a shape factor of 20 and a scale factor of 100.

1. What is the probability for a value equal to 105 ⇒ p(105) = ?
2. If the probability is 5%, what is the value ⇒ p<sup>-1</sup>(0.05) = ?
3. What is the probability of a value ≤ 90 ⇒ P(x ≤ 90) = ?
4. What is the value ‘z’ for probability of x ≤ z is 5% ⇒ P<sup>-1</sup>(x ≤ z) = 0.05 ?

### Solution:

Keystrokes	Description
Distribution [ Weibull ▶ ]	Select the Weibull Probability Distribution
20 [ k ], 100 [ λ ]	Input the shape ( k ) and scale ( λ ) parameters of the distribution.
105 [ p ( x ) ]	1) Calculate the probability. <b>Result = 0.035589</b>
0.05 [ p ( x ) <sup>-1</sup> ]	2) Calculate the z-value. <b>Result = 94.584178</b>
90 [ P ( x ) ]	3) Calculate the probability. <b>Result = 0.114477</b>
0.05 [ P ( x ) <sup>-1</sup> ]	4) Calculate the z-value. <b>Result = 86.199159</b>

# t-Student Probability Distribution



When the t-Student probability density function is selected, the distribution “Degrees of Freedom” parameter (DF) can be entered in the corresponding button.

The Probability Density Function is: 
$$p(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi} \Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

## Example: Weibull Distribution

Consider a t-Student random variable with 8 degrees of freedom.

1. What is the probability for a value equal to 0.5  $\Rightarrow p(0.5) = ?$
2. If the probability is 5%, what is the value  $\Rightarrow p^{-1}(0.05) = ?$
3. What is the probability of a value  $\leq 0.5$   $\Rightarrow P(x \leq 0.5) = ?$
4. What is the value ‘z’ for probability of  $x \leq z$  is 5%  $\Rightarrow P^{-1}(x \leq z) = 0.05 ?$

## Solution:

Keystrokes	Description
Distribution [ t-Student ► ]	Select the t-Student Probability Distribution
8 [ DF ]	Input the distribution degrees of freedom.
0.5 [ p(x) ]	1) Calculate the probability. <b>Result = 0.336694</b>
0.05 [ p(x) <sup>-1</sup> ]	2) Calculate the z-value. <b>Result = 2.145724</b>
0.5 [ P(x) ]	3) Calculate the probability. <b>Result = 0.684732</b>
0.05 [ P(x) <sup>-1</sup> ]	4) Calculate the z-value. <b>Result = -1.859548</b>