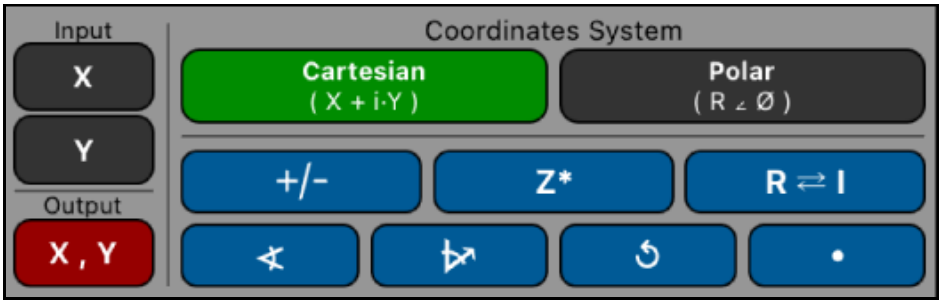
Complex Numbers Worksheet



This worksheet complement the Complex numbers operations of the calculator. A complex number is entered from the calculator using the “Input” buttons in the selected coordinate system (Cartesian or Polar).

Complex Worksheet Buttons	
[Cartesian]	Set Cartesian coordinates system.
Input: [X] [Y]	Input the calculator’s displayed number in the cartesian ‘X’ or ’Y’ coordinate.
Output: [X , Y]	Recalls to the calculator’s stack the corresponding coordinate (‘X’ -> stack-X and ‘Y’ -> stack-Y).
[Polar]	Set Polar coordinates system.
Input: [R] [Ø]	Input the calculator’s displayed number in: the radial distance ‘R’ to the origin or the polar angle ‘Ø’ (angle with respect to X-axis) coordinate.
Output: [R , Ø]	Recalls to the calculator’s stack the corresponding coordinate (‘R’ -> stack-X and ‘Ø’ -> stack-Y).
[+ / -]	Multiplies the **Zx** complex number by -1.
Z*	Conjugates **Zx** complex number (change the sign of the imaginary part).
[R ⇄ I]	Swaps the real and imaginary parts of **Zx** complex number.
[∢]	Calculates the angle between the **Zy** and **Zx** complex numbers.
[⊵]	Calculates the projection of **Zy** onto **Zx** complex number.
[↺]	Calculates the 90° counter-clock wise of **Zx** complex number.
[•]	Calculates the Dot product of **Zx** and **Zy** complex numbers.

When the Polar coordinates system is selected, the angles are entered and shown in the current angle unit.

To better understand how this worksheet works, follow the next examples carefully.

Example 1: (Arithmetic operations)
Evaluate the expression: [i·2 • (-8 + i·6)^3] / [(2 + i·3) • (4 + i·5)]

Solution:

Keystrokes	Description
[Cartesian]	Set the Cartesian coordinates.
0 **[X]** 2 **[Y]** **[ENTER]**	Enter the number "0 + i·2" -> **Zx = 0.00 + i·2.00**
8 **[CHS] [X]** 6 **[Y]** **[ENTER]**	Enter the complex number "-8 + i·6" -> **Zx = -8.00 + i·6.00**
3 **[X]** 0 **[Y]**	Enter the exponent number "3 + 0·i" -> **Zx = 3.00 + i·0.00**
[y ^{x}]	Calculate (-8 + 6·i)^3. Result: **Zx** = **352.00 + i·936.00**
[x]	Calculate 2·i•(-8 + 6·i)^3. Result: **Zx = -1,872.00 + i·704.00**
2 **[X]** 3 **[Y]** **[ENTER]**	Enter the complex number "2 + i·3" -> **Zx = 2.00 + i·3.00**
4 **[X] 5 [Y]**	Enter the complex number "4 + i·5" -> **Zx = 4.00 + i·5.00**
[x]	Calculates (2-i·3)•(4-i·5). Result: **Zx = -7.00 + i·22.00**
[÷]	Calculate the final result. Result: **Zx = 53.64 + i·68.02**
[X , Y]	Enters the imaginary part of **Zx** in stack-Y and the real part of **Zx** in stack-X.

Example 2: (Arithmetic operations)
Calculate the phasor expression: 2 ∠ 65° + 3 ∠ 40° and show the result in cartesian coordinates.

Solution: First, set DEG angular units pressing **[g] [DEG]**

Keystrokes	Description
[Polar]	Set Polar coordinates system.
2 [R] 65 [Ø] [ENTER]	Enter the 1st phasor -> **Zx = 2.00∠65.00**
3 [R] 40 [Ø]	Enter the 2nd phasor -> **Zx = 3.00∠40.00**
[+]	Adds the complex numbers phasors. Result: **Zx = 4.89∠49.96**
[Cartesian]	Set the Cartesian coordinates. Result: **Zx = 3.14 + i·3.74**

Example 3: (Parallel impedance)
Calculate total impedance of two parallel loads of 150 - i·106.1033 and 100 + i·24.5044.

Solution:

Keystrokes	Description
[Cartesian]	Set Polar coordinates system.
150 [X] 106.1033 [CHS] [Y]	Enter the 1st impedance -> **Zx = 150.00 - i·106.1033**
[1 / X]	Calculates the reciprocal -> **Zx = 0.0044 + i·0.0031**
100 [X] 24.5044 [Y]	Enter the 2nd impedance -> **Zx = 100.00 + i·24.5044**
[1 / X]	Calculates the reciprocal -> **Zx = 0.0094 - i·0.0023**
[+]	Adds the reciprocals -> **Zx = 0.0139 + i·0.0008**
[1 / X]	Total impedance -> **Zx = 71.8042 - i·4.3021**

Example 4:
Given Z_1 = -5+i·8 and Z_2 = 3 + i·4 calculate:

1) 3 x ($-Z_1$), conjugate the result and swap the imaginary and real parts.
2) Calculate the angle in degrees between Z_2 and Z_1
3) Get the projection of Z_2 over Z_1 and rotate it 90° counter clockwise.
4) Calculate the dot product of Z_1 and Z_2.

Solution: First, set DEG angular units pressing **[g] [DEG]**

Keystrokes	Description
[Cartesian]	Set Cartesian coordinates system.
1) 5 [CHS] [X] 8 [Y] [+/-] 3 [x] [Z*] [R⇄I]	Enter the Z_1 -> **Zx = -5.00 + i·8.00** Change sign and multiply by 3 -> **Zx = 15.00 - i·24.00** Conjugate Z_1 -> **Zx = 15.00 + i·24.00** Swap real and imaginary parts -> **Z1 = 24.00 + i·15.00**
2) 3 [X] 4 [Y] [ENTER] 5 [CHS] [X] 8 [Y] [∢]	Enter Z_2 -> **Zx = 3.00 + i·4.00** Enter the Z_1 -> **Zx = -5.00 + i·8.00** Angle between Z_1 and Z_2 -> **Zy∢Zx = -68.88**
3) [R↓] [⊵] [↺]	Recover Z_2 and Z_1 from previous operation. Projection of Z_2 over Z_1 -> **Zx = -0.96 + i·1.53** Rotate 90° counter-clock wise -> **Zx = -1.53 - i·0.96**
4) [g] [CLx] 5 [CHS] [X] 8 [Y] [•]	Clear Zx to prevent stack lifting (Z_2 is already in stack-Zy). Enter Z_1 again -> **Zx = -5.00 + i·8.00** Calculates the cosine -> **Zx•Zy = 17.00**

Example 5: (Trigonometric Functions)
Calculate all the trigonometric functions for Z = 3 + i·4

Solution:

Keystrokes	Description
[Cartesian]	Set Cartesian coordinates system.
3 [X] 4 [Y]	Enter the Z in polar coordinates -> **Zx = 3.00 + i·4.00**
[SIN]	Calculates the sine -> **Zx = 3.8537 - i·27.0168**
[g] [LSTx] [COS]	Calculates the cosine -> **Zx = -27.0349 - i·3.8512**
[g] [LSTx] [TAN]	Calculates the cosine -> **Zx = -0.0002 + i·0.9994**
[g] [LSTx] [g] [SIN^{-1}]	Calculates the $sine^{-1}$ -> **Zx = 0.6340 + i·2.3055**
[g] [LSTx] [g] [COS^{-1}]	Calculates the $cosine^{-1}$ -> **Zx = 0.9368 - i·2.3055**
[g] [LSTx] [g] [TAN^{-1}]	Calculates the $cosine^{-1}$ -> **Zx = 1.4483 + i·0.1590**

Example 6: (Hyperbolic Functions)
Calculate all the hyperbolic function for of Z = 1 + i·2

Solution:

Keystrokes	Description
[Cartesian]	Set Cartesian coordinates system.
1 [X] 2 [Y]	Enter the Z in polar coordinates -> **Zx = 1.00 + i·2.00**
[f] [HYP] [SIN]	Calculates the HYP sine -> **Zx = -0.4891 + i·1.4031**
[g] [LSTx] [f] [HYP] [COS]	Calculates the HYP cosine -> **Zx = -0.6421 + i·1.0686**
[g] [LSTx] [f] [HYP] [TAN]	Calculates the HYP cosine -> **Zx = 1.1667 - i·0.2435**
[g] [LSTx] [g] [HYP^{-1}] [SIN]	Calculates the HYP $sine^{-1}$ -> **Zx = 1.4694 + i·1.0634**
[g] [LSTx] [g] [HYP^{-1}] [COS]	Calculates the HYP $cosine^{-1}$ -> **Zx = 1.5286 + i·1.1437**
[g] [LSTx] [g] [HYP^{-1}] [TAN]	Calculates the HYP $cosine^{-1}$ -> **Zx = 0.1733 + i·1.1781**