## Spherical Triangle Menu



This menu allows you to solve a spherical triangle in the surface of sphere of default radius 1 .

Depending on the calculus mode (AAA, AAS, ASA, SAS, SSA or SSS) the menu dynamically changes indicating, what variables are inputs (Black colored buttons), and what variables are calculated (Red colored Buttons).

Each time a value is entered in an input variable by touching a black colored button, all the calculations are performed at once. If an input value results in an invalid triangle with no solution, "---" is shown as the calculated values.

In general, a triangle has a solution if some basic conditions are met:

- The length of sides ' $\mathbf{a}$ ', ' $\mathbf{b}$ ' and ' $\mathbf{c}$ ' must be greater than 0 .
- The sum of any two sides must be greater than the other side.
- The angles ' $A$ ', ' $B$ ' and ' $C$ ' must be greater than 0 .
- The angles ' $A$ ', ' $B$ ' and ' $C$ ' must be less than $\boldsymbol{\pi}$ ( $180^{\circ}$ )
- The sum of any two angles must be less than $\boldsymbol{\pi}\left(180^{\circ}\right)$.

The following table summarize all the possibilities of the input and output buttons depending in the selected calculation mode:

| Mode | Button | Description |
| :---: | :---: | :---: |
| [ AAA ${ }^{\text {] }}$ ] | [ Side ' $a$ '] <br> [ Side 'b' ] <br> [ Side 'c'] <br> [Angle 'A'] <br> [Angle ' $B$ '] <br> [Angle ' $C$ '] | Calculates the side 'a' length. Calculates the side ' $\mathbf{b}$ ' length. Calculates the side ' $\mathbf{c}$ ' length. Stores angle ' $A$ ' in current angle unit. Stores angle ' $\mathbf{B}$ ' in current angle unit. Stores angle ' $\mathbf{C}$ ' in current angle unit. |
| [ AAS ${ }^{\text {] }}$ ] | [ Side ' $a$ '] <br> [ Side 'b' ] <br> [ Side 'c'] <br> [Angle 'A'] <br> [Angle ' $B$ '] <br> [Angle 'C'] | Calculates the side ' $a$ ' length. <br> Stores side 'b' length. <br> Calculates the side ' $\mathbf{c}$ ' length. <br> Stores angle ' $A$ ' in current angle unit. <br> Stores angle ' $B$ ' in current angle unit. <br> Calculates the angle ' $\mathbf{C}$ '. |
| [ ASA $>$ ] | [ Side 'a'] <br> [ Side 'b' ] <br> [ Side ' $c$ '] <br> [Angle 'A'] <br> [Angle ' $B$ '] <br> [Angle 'C'] | Calculates the side ' $a$ ' length. Calculates the side 'b' length. Stores side ' $\mathbf{c}$ ' length. <br> Stores angle ' $A$ ' in current angle unit. Stores angle ' $B$ ' in current angle unit. Calculates the angle ' $\mathbf{C}$ '. |
| [ SAS ${ }^{\text {] }}$ ] | [ Side 'a'] <br> [ Side 'b' ] <br> [ Side 'c'] <br> [Angle ' $A$ '] <br> [Angle ' $B$ '] <br> [Angle ' $C$ '] | Stores side 'a' length. <br> Stores side 'b' length. <br> Calculates the side ' $\mathbf{c}$ ' length. <br> Calculates the angle ' $\mathbf{A}$ '. <br> Calculates the angle ' $B$ '. <br> Stores angle ' $\mathbf{C}$ ' in current angle unit. |
| [ SSA $>$ ] | $\begin{aligned} & \text { [ Side 'a' ] } \\ & \text { [ Side 'b'] } \\ & \text { [ Side 'c'] } \\ & \text { [Angle 'A'] } \\ & \text { [Angle 'B'] } \\ & \text { [Angle 'C'] } \end{aligned}$ | Calculates the side ' $a$ ' length. <br> Stores side 'b' length. <br> Stores side ' $\mathbf{c}$ ' length. <br> Calculates the angle ' $\mathbf{A}$ '. <br> Stores angle ' $\mathbf{B}$ ' in current angle unit. <br> Stores angle ' $\mathbf{C}$ ' in current angle unit. |
| [ SSS ${ }^{\text {] }}$ ] | $\begin{aligned} & \text { [ Side 'a' ] } \\ & \text { [ Side 'b'] } \\ & \text { [ Side 'c'] } \\ & \text { [Angle 'A'] } \\ & \text { [Angle 'B'] } \\ & \text { [Angle 'C'] } \end{aligned}$ | Stores side 'a' length. Stores side 'b' length. Stores side ' $\mathbf{c}$ ' length. Calculates the angle ' $\mathbf{A}$ '. Calculates the angle ' $B$ '. Calculates the angle ' $\mathbf{C}$ '. |
| [ Radius ] |  | Stores Radius of the sphere where the triangle |
| [ Perimeter ] |  | Calculates the spherical triangle Perimeter. |
| [ Area ] |  | Calculates the spherical triangle Area. |

## Example: (AAA)

Given a spherical triangle with interior angles of $89^{\circ}, 66^{\circ}$ and $70^{\circ}$, what is the triangle area and perimeter if it is in a sphere of radius 1000 meters?.

## Solution: ( DEG angle mode)

| Keystrokes | Description |
| :---: | :--- |
| [ AAA ] | Set 'AAA' calculation mode. |
| 89 [ Angle 'A' ] | Store the angle 'A'. |
| 66 [ Angle 'B'] | Store the angle 'B'. |
| 70 [ Angle 'C'] | Store the angle 'C'. |
| 1000 [ R ] | Stores the sphere radius. |
| [ Perimeter ] | The perimeter of the spherical triangle is $3,682.02 \mathrm{~m}$ |
| [ Area ] | The area of the triangle is $\mathbf{7 8 5 , 3 9 8 . 1 6 ~} \mathrm{m}^{2}$ |

## Example: (SSS)

In a sphere of 100 cm radius is a surface triangle $A B C$, the sides are $6 \mathrm{~cm}, 10 \mathrm{~cm}$ and 14 cm . Show that the triangle is obtuse angled with the obtuse angle equal to $120^{\circ}$.

## Solution: ( DEG angle mode)

| Keystrokes | Description |
| :---: | :---: |
| [ SSS - ] | Set 'SSS' calculation mode. |
| 100 [ Radius ] | Enter the sphere radius. |
| $\begin{gathered} 6 \text { [ Side 'a'] } \\ 10 \text { [ Side 'b'] } \\ 14 \text { [ Side 'c'] } \end{gathered}$ | Store the ' $a$ ' side length. Store the 'b' side length. Store the ' $c$ ' side length. |
| [ Angle 'C'] | The Angle ' $C$ ' is $120.05^{\circ}$ which is greater than $90^{\circ}$. |

## Example: (SAS)

Two sides of a triangle, in the surface of sphere of radius 500 cm , are 78 and 56 cm and their included angle is $60^{\circ}$. What is the triangle's area and perimeter?

## Solution: ( DEG angle mode)

| Keystrokes | Description |
| :---: | :---: |
| [ SAS -] | Set 'SAS' calculation mode. |
| 500 [Radius ] | Enter the sphere radius. |
| $\begin{gathered} 78 \text { [ Side 'a'] } \\ 56 \text { [ Side 'b'] } \\ 60 \text { [ Angle 'C'] } \end{gathered}$ | Store the 'a' side length. <br> Store the 'b' side length. <br> Store the ' $C$ ' angle in degrees. |
| [ Perimeter] | The perimeter of the triangle is 203.52 cm |
| [ Area ] | The area of the triangle is $1,893.07 \mathrm{~cm}^{2}$ |

