## 3D Vectors Worksheet

| $\begin{gathered} \hline \text { Input } \\ \mathrm{X} \\ \hline \end{gathered}$ | Coordinates System |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cartesian （X，Y，Z） |  |  | Cylindrical （P， $\mathrm{\theta}, \mathrm{Z}$ ） |  |
| $Y$ | $\|\mathrm{V}\|$ | 丈 | $b$ | û | r |
| Z | ＋／－ | K | － | ＊ | Z |

This worksheet implements a 3D Vector stack to perform operations and functions over it．The＂3D Vector Stack＂is similar to the normal calculator＇s stack， but specially designed for operations with vectors．The vector coordinates components are entered from the calculator using the＂Input＂buttons in the selected coordinates system（Cartesian，Spherical or Cylindrical）．

| 3D Vector Menu Actions |  |
| :---: | :---: |
| $\begin{gathered} \text { [ Cartesian ] } \\ \text { Input: } \\ \text { [ X ] [ Y ] [ Z ] } \\ \text { Output: } \\ \text { [ X ] [ Y ] [ }] \end{gathered}$ | Set Cartesian coordinates system． Input the calculator＇s displayed number in the cartesian＇$X$＇，＇$Y$＇or＇$Z$＇ coordinate． <br> Recalls to the calculator the corresponding＇$X$＇，＇$Y$＇or＇$Z$＇coordinate． |
| ［Spherical］ <br> Input： <br> ［R］［日］［0］ <br> Output： <br> ［ R ］［ O ］［ O ］ | Set Spherical coordinates system． <br> Input the calculator＇s displayed number in the radial distance＇$R$＇to the origin，the polar angle＇$\Theta$＇（angle with respect to X －axis in $\mathrm{X}-\mathrm{Y}$ plane）or the＇$\varnothing$＇angle between $Z$－axis and the line from the origin to the point． <br> Recalls to the calculator the corresponding＇$R$＇，＇$\Theta$＇or＇$\varnothing$＇coordinate． |
| ［ Cylindrical］ <br> Input： $\text { [ P ] [ } \boldsymbol{\theta} \text { ] [ Z ] }$ <br> Output： [P][日][Z] | Set Cylindrical coordinates system． <br> Input the calculator＇s displayed number in the polar distance＇$P$＇in the $X-Y$ plane，the angle＇$\Theta$＇（angle with respect to $X$－axis in $X-Y$ plane）or the＇$Z$＇coordinate． <br> Recalls to the calculator the corresponding＇$P$＇，＇$\Theta$＇or＇$Z$＇coordinate． |


| 3D Vector Menu Actions |  |
| :---: | :---: |
| [ I V \| ] | Calculates the Magnitude of the Vx vector. |
| [ ${ }^{\text {] }}$ | Calculates the angle between the Vx and Vy vectors. |
| [ $\dagger$ ] | Calculates the projection of vector $\mathbf{V y}$ onto vector Vx. |
| [ û ] | Calculates the unitary vector of $\mathbf{V x}$ (same direction with magnitude 1.0). |
| [ + / - ] | Multiplies the vector Vx by $\mathbf{- 1}$. |
| [ K ] | Scales the Vx vector by the calculator's stack-X value. |
| [•] | Calculates the Dot product of $\mathbf{V x}$ and $\mathbf{V y}$ vectors and enters the result in the calculator's display. |
| [ ® ] | Calculates the Cross product of $\mathbf{V x}$ and $\mathbf{V y}$. Drop the vector stack and put the result in $\mathbf{V x}$. |

To operate or manipulate the 3D vector stack, use the keys ' $\mathrm{X} \rightleftarrows \mathrm{Y}$ ', ' $\mathrm{R} \downarrow$ ', ' $R \uparrow$ ', ‘CL $X$ ', 'ENTER', 'LST $X$, ‘’’ or ‘-‘ available in the calculator's keyboard.

When the Spherical or Cylindrical coordinates system is selected, the angles are shown in the current angle unit setting. Nevertheless, internally, all the 3D vectors are stored in Cartesian coordinates ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ).

To better understand how this menu works, follow the next examples carefully.

Example 1: (Scale, Magnitude and different coordinates system)
Considering the vector $(3,4,5)$ in cartesian coordinates:

1. Scale by 3 and Express the result in spherical coordinates.
2. Calculate de magnitude of the result.
3. Scale the original vector by 0.5 and express it in cylindrical coordinates

## Solution ( DEG angle unit ) :

| [ Cartesian ] | Set Cartesian coordinates. |
| :---: | :---: |
| $3[\mathbf{X}] 4$ [ Y ] 5 [ Z ] | Input the vector A : $\operatorname{Vx}(3.00,4.00,5.00)$ |
| 3 [ K ] | Type 3 and scale the vector: Result: Vx(9.00, 12.00, 15.00) |
| [ Spherical ] | 1) Set Spherical coordinates. Result: Vx(21.21, $\llcorner 53.13, ~<45.00)$ |
| [ I V I] | 2) Calculate the magnitude. Result: $\mathbf{I V x I}=21.21$ |
| ```[g][LSTx] 0.5 [ K ] [ Cylindrical ]``` | Recuperate original vector. Type 0.5 and scale the vector: <br> 3) Set Cylindrical coordinates. <br> Result: Vx(2.50, $\angle 53.13,2.50)$ |

Example 2: (Angle and Projection)
Given vector-A $=(3,-2,5)$ in cartesian coordinates and vector-B $=\left(15, \angle 25^{\circ}\right.$, $\angle 42^{\circ}$ ) in spherical coordinates, find:

1) The angle between them.
2) The projection of vector-B onto vector-A in cartesian coordinates.

## Solution ( DEG angle unit ) :

| [ Spherical ] | Set Spherical coordinates |
| :---: | :---: |
| $\begin{gathered} 15 \text { [ R ] } 25 \text { [CHS] [ © ] } 42 \text { [ Ø ] } \\ \text { [ ENTER ] } \end{gathered}$ | Input vector-B $\operatorname{Vx}(15.00,<-25.00,<42.00)$ |
| [ Cartesian ] | Set Cartesian coordinates. |
| 3 [ X ] 2 [CHS] [ Y ] 5 [ Z ] | Input vector-A $V x(3.00,-2.00,5.00)$ |
| [ * ] |  |
| [ $\downarrow$ ] | 2) Projection of vector-B in vector-A. Result: $\mathbf{V x}(7.22$, -4.82, 12.04) |

Example 3: (Minus and Unitary vector)
A vector $A B$ is directed from point $A(-1,-2,1)$ to point $B(-2,3,4)$, find the the unit vector of the $A B$.

## Solution:

| Keystrokes | Description |
| :---: | :--- |
| [ Cartesian ] | Set Cartesian coordinates. |
| 2 [CHS] [ X ] 3 [ Y ] 4 [ Z ] |  |
| [ ENTER ] |  |\(\left.\quad \begin{array}{l}Input vector-B <br>


Vx(-2.00, 3.00, 4.00)\end{array}\right]\)| Input vector-A |
| :--- |
| Vx(-1.00, -2.00, 1.00) |

Example 4: (Add and cross product)
Add a vector- $\mathrm{A}=\left(5,60^{\circ}, 45^{\circ}\right)$ in spherical coordinates to a vector- $\mathrm{B}=\left(8,22^{\circ}, 3\right)$ in cylindrical coordinates. Then calculate the cross product with a cartesian vector-C $=(0.5,0.34,0.25)$. Show the results in cartesian coordinates :
Solution ( DEG angle unit ):

| Keystrokes | Description |
| :---: | :---: |
| [ Spherical ] | Set Spherical coordinates |
| 5 [ R ] 60 [ $\boldsymbol{0}$ ] 45 [ 0 ] | $\begin{aligned} & \text { Input vector-A } \\ & \mathrm{Vx}(5.00, \angle 60.00, \angle 45.00) \end{aligned}$ |
| [ Cylindrical ] | Set Cylindrical coordinates |
| 8[P]22[日] 3 [ $\mathbf{~ ] ~}$ | Input vector-B $\operatorname{Vx}(8.00, \angle 22.00,3.00)$ |
| [ + ] | Add Vx and Vy. Result: Vx(11.00, $\llcorner\mathbf{3 3 . 4 1 , ~ 6 . 5 4 )}$ |
| [ Cartesian ] | Set Cartesian coordinates. Result: Vx(9.19, 6.06, 6.54) |
| $\begin{gathered} 0.5[\mathbf{X}] 0.34[\mathbf{Y}] 0.25[\mathbf{Z}] \\ {[\mathbb{\otimes}]} \end{gathered}$ | Input vector-C. $\mathrm{Vx}(0.50,0.34,0.25)$ Cross product. Result: Vx(-0.71, 0.97, 0.09) |

